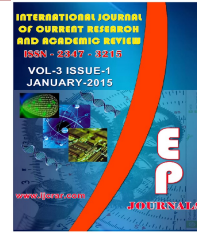




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### Comparative Study of Stress Analysis on Notched Plates between Analytical and Finite Element Solutions

K. S. Babulal\*, I. Vimal Kannan and A.Pradeep

Department of Mechanical Engineering, Care Group of Institutions, Tiruchirappalli, India

\*Corresponding author

#### KEYWORDS

Finite Element Method,  
Analytical Method,  
Notched Plate,  
Static Analysis.

#### A B S T R A C T

Notched plates of various shapes find its applications in steel structural and machine elements. Failure of the notched element occurs under the loading condition in the region where the stress distribution is disturbed. The proper study of effect of shapes and orientations of notches is imperative. The prediction of maximum stress developed at notches avoids such failures that ensure safe design of elements. The present work is to envisage the maximum stress developed at three different types of notches in steel plates namely; semi-circular, U-shaped and V-shaped notches and the obtained analytical solutions for maximum stresses are found to be 150.88 MPa, 167.35 MPa and 201.09 MPa respectively. The analytical results have been compared with finite element method using SolidWorks Simulation Software. The static analysis shows that the percentage of error between analytical and finite element solutions has been found to be less than 2%.

### Introduction

In developing many structural elements and machines, it is impossible to avoid changes in cross-section, holes, notches, grooves etc. in order to reduce weight or provide access to the other part of the structures or machines (Pilkey and Peterson, 1997). In such cases, the distributions of stress in which the peak stress reaches would be very large in magnitude than the average stress over the section. Various shapes of notched plates are frequently employed in a range of applications from microelectronic devices to large scale civil structures (Andrei Kotousov, 2010). The presence of hole or notch in plates introduces highly localized stresses at

the environs of the notches, it is important to consider stress concentration and the factors responsible at the design stage. It is possible to predict the stress concentration factors (SCF) for certain geometric shapes using theory of elasticity approach (Timoshenko and Goodier, 1970; Neuber, 1961).

Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at points of stress concentration. The degree of stress and strain concentration is a factor in the fatigue

strength of notched parts. It is measured by the elastic stress concentration factor ( $K_t$ ) for static load. It can be determined as the relation of the actual maximum real stress in the discontinuity and the average stress (Hamrocket *al.*, 2013). The average stress is determined by the ratio of axial force to the cross sectional area and is defined according to the type of load that is acting on the element. In the case of an axial load that causes tension or compression (Gere and Goodno, 2008).

Many researches have been performed on stress concentration factor in isotropic, anisotropic and composite materials (Konish and Whitney, 1975; Emery *et al.*, 2008; Awerbuch and Madhukar, 1985; Lotfiet *al.*, 2005; Lekhnitskii, 1967) with circular and elliptical cut outs (Savin, 1961; Dheeraj Gunwant and Singh, 2013). Investigation on the SCF for isotropic plates under uni-axial and bi axial loads has been performed (Wu and Mu, 2003). SCF of a round bar with a circular-arc or V-shaped notch are considered under torsion, tension and bending (Nao-Aki Noda and Yasushi Takase, 2006). The finite element method is an alternative procedure to analytical methods for analyzing structural elements in which continuous interconnected by a series of points called nodes. The equations governing the behaviour of continuous also govern the systems are discretized into a smaller set of elements element. Thus it is achieved to pass from a continuous system (infinite degrees of freedom), which is governed by a differential equation or a system of differential equations to a system with a number of finite degrees of freedom whose behaviour is modeled by a system of linear or non-linear equations (Chandrupatla

and Belegundu, 2002; Zienkiewicz and Taylor, 2013). Less research has been focused on stress concentration factor on notched plate. In the present era, with the development in FEA based softwares, stress analysis is accurate. According to the type of stress analysis and type of elements to be analyzed, the FEA tool is chosen. SolidWorks simulation plays an essential role in both static and dynamic analysis stress in the recent years. In the present work, the modeling and static analysis of steel plates has been performed with semi-circular, U-shaped and V-shaped notches.

## Experimental Method

### Analytical Method

In this present study, stress concentration factor (SCF) of notched plate of size 250 mm × 250 mm × 25 mm is used. Three different types of notches namely; semi-circular, U-shaped and V-shaped notches, were analysed under axial tensile pressure of 50 N/mm<sup>2</sup>. The average stress and stress concentration factor  $K_t$  has been calculated by using the equation (1) and (2) respectively,

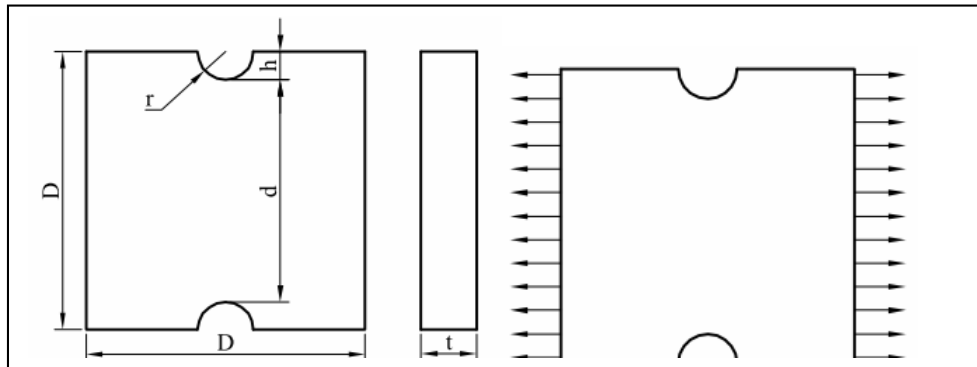
$$\sigma = \frac{\text{Axial Force}}{\text{Cross Sectional Area}} \quad \text{--- (1)}$$

$$K_t = \frac{\text{Actual Maximum Stress}}{\text{Average Stress}} \quad \text{--- (2)}$$

### Semi-Circular Notch: $\left(\frac{h}{r} = 1\right)$

Fig.1 shows plate with opposite single semi-circular shaped notches. Dimensions for the notched plate are depicted in Table.1.

**Fig.1** Opposite single semi-circular shaped notches



**Table.1** Dimensions for the semi – circular notched plate

r	= 25 mm
h	= 25 mm
t	= 25 mm
D	= 250 mm
d	= (D - 2r) = 200 mm
P	= p × D × t = 312500 N

**Calculation:**

$$K_t = 3.065 - 3.472 \left(\frac{2h}{D}\right) + 1.009 \left(\frac{2h}{D}\right)^2 + 0.405 \left(\frac{2h}{D}\right)^3 ;$$

$$K_t = 2.414.$$

$$\sigma_{nom} = \frac{P}{td} = 62.5 \text{ N / mm}^2;$$

$$\sigma_{max} = \sigma_A = K_t \sigma_{nom} = 150.875 \text{ N / mm}^2.$$

**U – Shaped Notch:  $\left(\frac{h}{r} = 1.6\right)$**

Fig.2 shows plate with opposite single U–Shaped notches. Dimensions for the notched plate are depicted in Table.2. Constants used in U-shaped notches depend on the range of  $\left(\frac{h}{r}\right)$  ratio and were depicted in the Table.3.

Fig.2 Opposite single U-Shaped notches

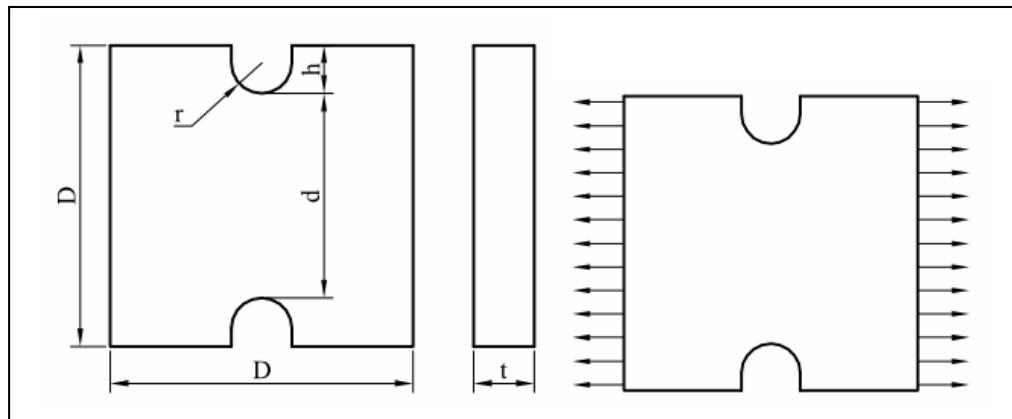


Table.2 Dimensions for the U - shaped notched plate

r	= 25 mm
h	= 40 mm
t	= 25 mm
D	= 250 mm
d	= (D - 2h) = 170 mm
P	= p × D × t = 312500 N
C <sub>1</sub>	= 3.5694
C <sub>2</sub>	= - 5.0228
C <sub>3</sub>	= 3.3522
C <sub>4</sub>	= - 0.8936

Table.3 Constants used in U-shaped notches

C VALUES	$0.1 \leq \frac{h}{r} < 2.0$	$2.0 \leq \frac{h}{r} \leq 50.0$
C <sub>1</sub>	$0.955 + 2.169 \sqrt{\frac{h}{r}} + 0.081 \left(\frac{h}{r}\right)$	$1.037 + 1.991 \sqrt{\frac{h}{r}} + 0.002 \left(\frac{h}{r}\right)$
C <sub>2</sub>	$-1.557 - 4.046 \sqrt{\frac{h}{r}} + 1.032 \left(\frac{h}{r}\right)$	$-1.886 - 2.181 \sqrt{\frac{h}{r}} - 0.048 \left(\frac{h}{r}\right)$
C <sub>3</sub>	$4.013 + 0.424 \sqrt{\frac{h}{r}} - 0.748 \left(\frac{h}{r}\right)$	$0.649 + 1.086 \sqrt{\frac{h}{r}} + 0.142 \left(\frac{h}{r}\right)$
C <sub>4</sub>	$-2.461 + 1.538 \sqrt{\frac{h}{r}} - 0.236 \left(\frac{h}{r}\right)$	$1.218 - 0.922 \sqrt{\frac{h}{r}} - 0.086 \left(\frac{h}{r}\right)$

**Calculation:**

$$K_t = C_1 + C_2 \left(\frac{2h}{D}\right) + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3 ;$$

$$K_t = 2.276.$$

$$\sigma_{nom} = \frac{P}{td} = 73.53 \text{ N/mm}^2;$$

$$\sigma_{max} = \sigma_A = K_t \sigma_{nom} = 167.354 \text{ N/mm}^2.$$

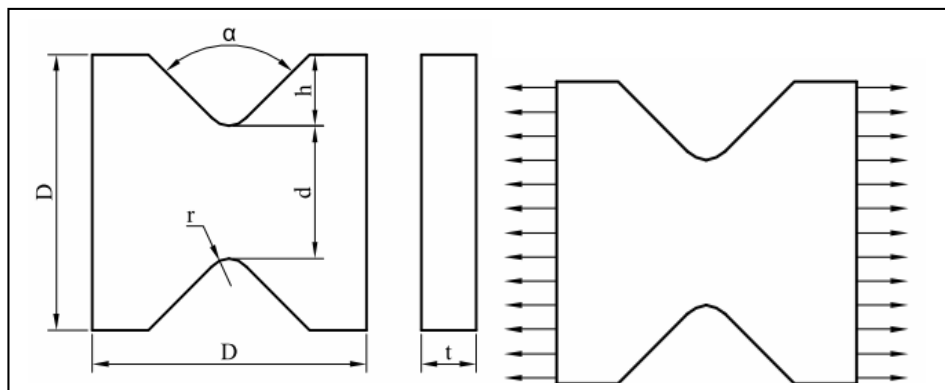
**V – Shaped Notch:  $\left(\frac{2h}{D} = 0.398 ; \alpha = 120^\circ ; K_{tu} = 2.55\right)$**

Fig.3 shows plate with opposite single V–Shaped notches. Dimensions for the notched plate are depicted in Table.4. The different values of C to be calculated for the range of  $\left(\frac{2h}{D}\right)$  ratio and  $\alpha$  and  $K_{tu}$  values are depicted in Table.5.

**Table.4** Dimensions for the V - shaped notched plate

r	= 25 mm
h	= 49.75 mm
t	= 25 mm
D	= 250 mm
d	= (D – 2h) = 150.5 mm
P	= p × D × t = 312500 N
C <sub>1</sub>	= -1.875
C <sub>2</sub>	= 2.803
C <sub>3</sub>	= -0.0706

**Fig.3** Opposite single V–Shaped notches



**Table.5** Constants used in V-shaped notches

C VALUES	$\left(\frac{2h}{D}\right) = 0.398, 90^0 \leq \alpha \leq 150^0,$ $1.6 \leq K_{tu} \leq 3.5$	$\left(\frac{2h}{D}\right) = 0.667, 60^0 \leq \alpha \leq 150^0,$ $1.6 \leq K_{tu} \leq 2.8$
C <sub>1</sub>	$5.294 - 0.1225 \alpha + 0.000523 \alpha^2$	$-10.01 + 0.1534 \alpha - 0.000647 \alpha^2$
C <sub>2</sub>	$-5.0002 + 0.1171 \alpha - 0.000434 \alpha^2$	$13.60 - 0.2140 \alpha + 0.000973 \alpha^2$
C <sub>3</sub>	$1.423 - 0.01197 \alpha - 0.000004 \alpha^2$	$-3.781 + 0.07873 \alpha - 0.000392 \alpha^2$

**Calculation**

$$K_t = C_1 + C_2 \sqrt{K_{tu}} + C_3 K_{tu};$$

$$K_t = 2.421.$$

$$\sigma_{nom} = \frac{P}{td} = 83.06 \text{ N / mm}^2;$$

$$\sigma_{max} = \sigma_A = K_t \sigma_{nom} = 201.089 \text{ N / mm}^2.$$

**Finite Element Method**

Notched plates have been modelled using Solidworks part modelling and the static analysis has been done using simulation. The dimensions of notched plates are 250 mm (x-direction), 250 mm (y-direction), and 25 mm (z-direction). The value of ‘h’ is different for each type of notches. Fixtures are applied to prevent out of plane rotations and free body motions.

The plate is fixed in the mid – plane. The model will become stable when edges and a vertex, which is not a part of edges, are fixed. To restrain the translation motion in the direction of ‘x’, six vertical edges are fixed along the front plane direction. To restrain the translation motion in the direction of ‘y’, six horizontal edges are fixed along the front plane direction. Finally vertex at the right bottom corner is fixed normal to front plane so as to restrain motion in the direction of ‘z’. The loading pressure is 50 MPa, which is to be applied at both ends of the vertical faces. It has the same magnitude, but opposite in directions.

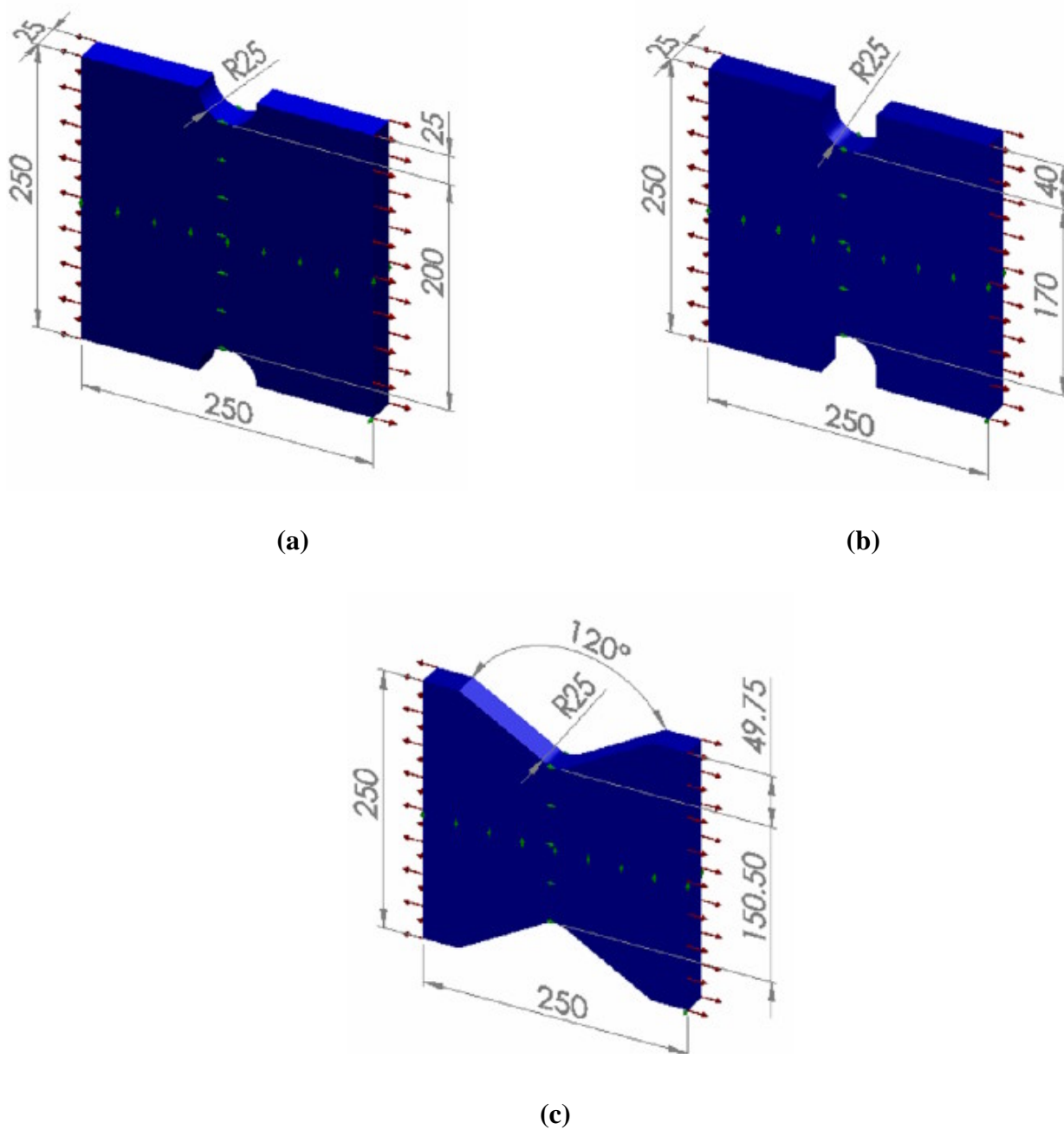
3D model for semi-circular, U-shaped and V-shaped notches are shown in Fig.4 (a), (b) and (c) respectively.

**Result and Discussion**

The maximum normal stress and stress concentration factor have been evaluated using SolidWorks simulation by considering different shape of notches with different variables. These results are given as follows,

**Static nodal stress analysis for semi – circular notch**

In the static stress analysis, the size of the plate is kept constant for all type of notch,  $\left(\frac{h}{r}\right)$  ratio differs in which the value of ‘r’ is kept constant as 25 mm and ‘h’ values vary according to the type of notches. In case of semi – circular notch  $\left(\frac{h}{r} = 1\right)$  and hence, h = 25 mm. From the static analysis, as expected, the maximum normal stress occurs at the centre of the semi – circular notched portion. The maximum stress is 153.75 MPa, shown in Fig.5 and the stress concentration factor results in 2.46.



**Fig.4** 3D model for (a) semi-circular, (b) U-shaped and (c) V-shaped notches.

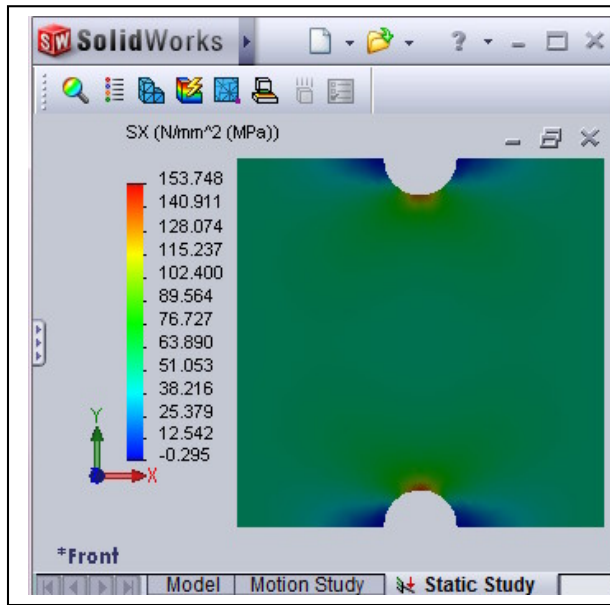


Fig.5 Maximum Normal Stress – Semi circular notch

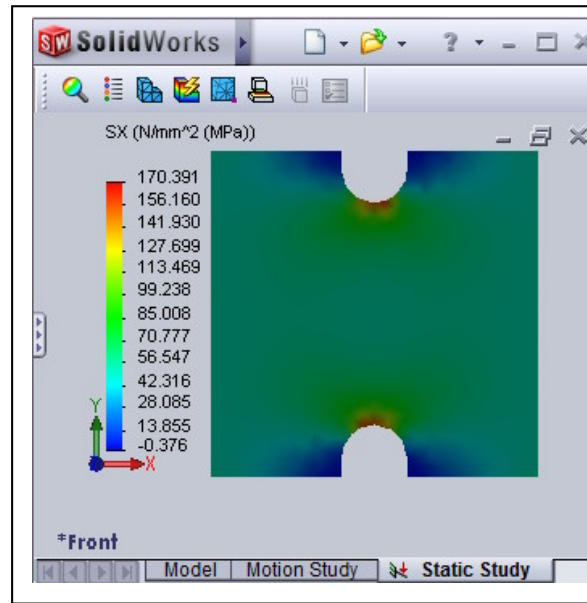


Fig.6 Maximum Normal Stress – U notch

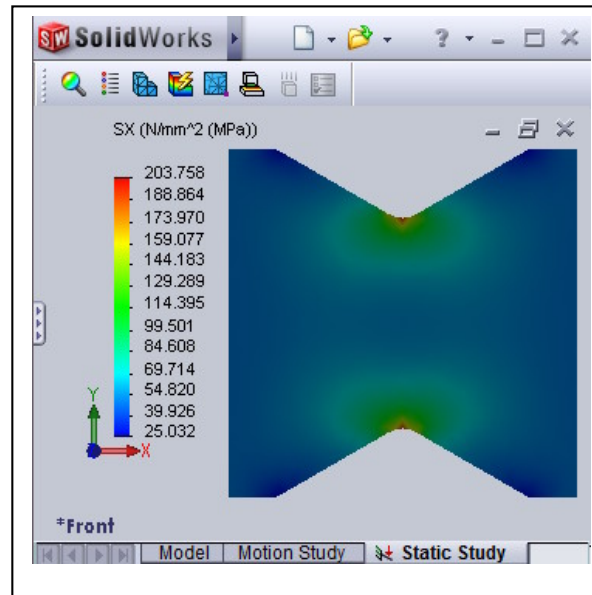


Fig.7 Maximum normal stress – V – notch



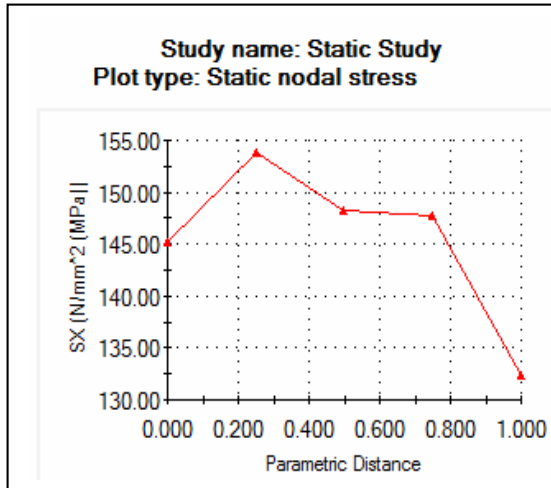


Fig.8 Distribution of stress at semi circular notch

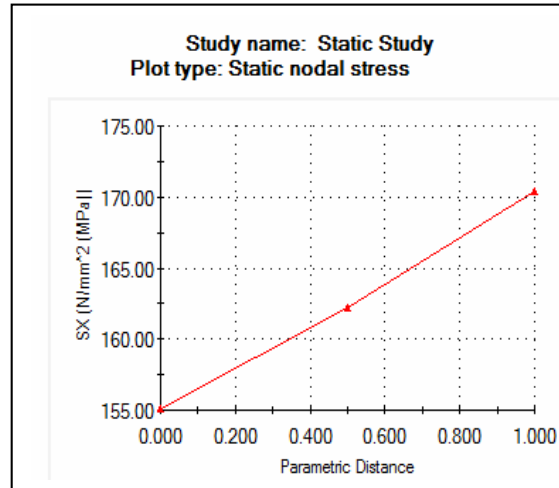


Fig.9 Distribution of stress at U- notch

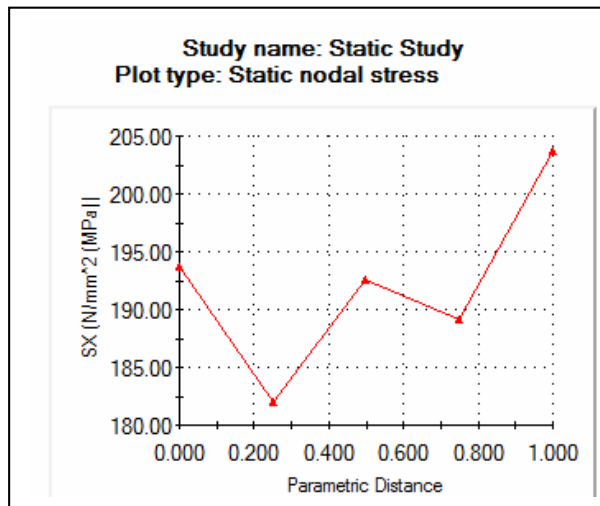


Fig.10 Distribution of stress at V- notch

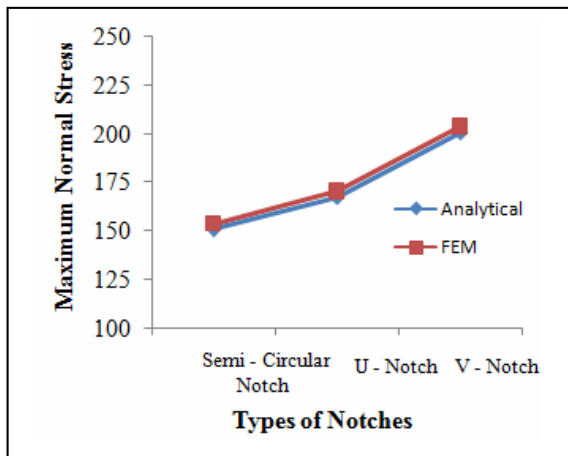


Fig.11 Comparison of maximum normal stress between analytical and FEM analysis

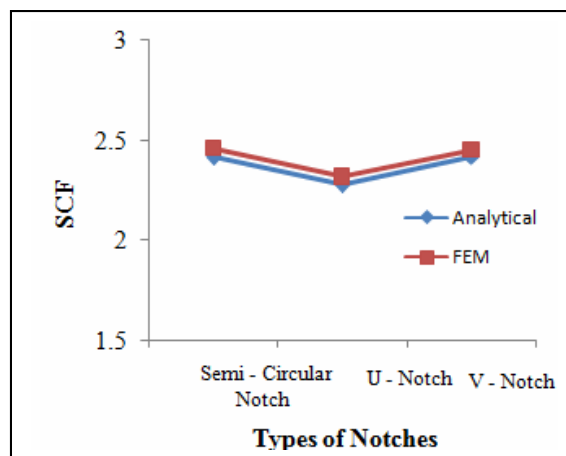


Fig.12 Comparison of SCF between analytical and FEM analysis

**Table.6** Percentage of error calculation between analytical and FEM results

S.No.	Type of Stress Raiser	Analytical Solution ( $\sigma_{max}$ ) MPa	Finite Element Solution ( $S_{max}$ ) MPa	Percentage of Error $\left(\frac{\sigma_{max} - S_{max}}{\sigma_{max}}\right) \times 100$ (%)
1	Opposite Single Semicircular – Notch	150.875	153.748	1.90
2	Opposite Single U – Shaped Notch	167.354	170.391	1.81
3	Opposite Single V – Shaped Notch	201.089	203.758	1.33

**Static nodal stress analysis for U – notch**

In case of U – notch ( $\frac{h}{r} = 1.6$ ) and hence, h = 40 mm. From the static analysis, as expected, the maximum normal stress occurs at the centre of the U – notched portion. The maximum stress is 170.391 MPa, shown in Fig.6 and the stress concentration factor results in 2.31.

**Static nodal stress analysis for V – notch**

In case of V – notch, average of notch angle  $\alpha$  is taken as  $120^\circ$  ( $\frac{2h}{D} = 0.398$ ) and hence, h = 49.75 mm. Also Stress concentration factor for U-shaped notch,  $K_{tu} = 2.5$ . From the static analysis, the maximum normal stress occurs at the centre of the V – notched portion. The maximum stress is 203.758 MPa, shown in Fig.7 and the stress concentration factor results in 2.45.

The distribution of stress with respect to normalized thickness (parametric distance) of plate for semi circular, U-notch and V-notch is shown in Fig. 8, 9 and 10 respectively. In semi – circular notch, the

initial stress decreases from 145.275 MPa to 132.419 MPa, after reaching its maximum stress.

The distribution of stress in U – notch and V – notch shows that the initial stress increases from 155.140 MPa to 170.391 MPa and 193.721 MPa to 203.758 MPa, respectively. The variation of maximum stress between analytical and finite element method for maximum stress and the corresponding SCF is plotted in Fig.11 and 12 respectively. The percentage of error falls below 2 % which is acceptable and the results are tabulated in Table.6.

**Conclusion**

The results are obtained for maximum normal stress and SCF through analytical solution and finite element method. The following conclusions are drawn to the present study.

1. Shape of the notch is significant in the magnitude of maximum stresses developed in the notched plate.
2. In case of semi – circular notch, the maximum normal stress is about three times the applied load. For U –

Shaped notch it is more than three times of applied load. For V – Shaped notch it is about four times of applied load.

3. The maximum stress concentration occurs at centre of notched plate irrespective of notch shapes.
4. The percentage of error between analytical and finite element solutions for all types of notches is less than 2 %, which lies in the acceptable limit.

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